

# Geometry SOL Review

# Study Guide of Important Information

## G.1 Logic

Conditional	Converse	Inverse	Contrapositive	Biconditional
$p \rightarrow q$	$q \rightarrow p$	$\sim p \rightarrow \sim q$	$\sim q \rightarrow \sim p$	$p \leftrightarrow q$

- Contrapositive is true when the conditional is true.
- Converse and inverse have the same truth value
- Additional symbols: "and" "or"

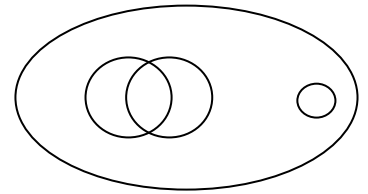
## Law of Detachment

- \*one conditional statement
- \*second statement sounds like first statement hypothesis
- \*conclusion sounds like first conclusion

## Law of Syllogism

- \*two conditional statements
- \*first conclusion repeats as second hypothesis
- \*conclusion is: If (1<sup>st</sup> hypothesis then (2<sup>nd</sup> conclusion)

## Venn Diagram



- \*all small in large
- \*some large in small
- \*some of each overlap in the other
- \*none when no overlap

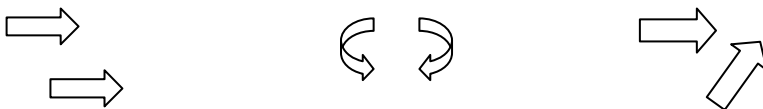
## G.2 Coordinate Formulas and Transformations

### Formulas:

Midpoint	Distance	Slope
$(\frac{x_2+x_1}{2}, \frac{y_2+y_1}{2})$	$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	$\frac{y_2-y_1}{x_2-x_1}$

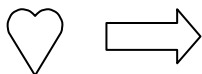
### Transformations:

Translation (slide) Reflection (flip/fold) Rotation (spin/turn)

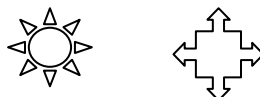


### Symmetry:

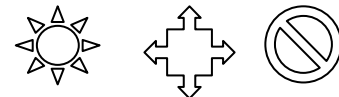
Line  
– fold line; folds figure exactly in half, one half onto the other



Rotational  
– spin figure by a degree value and figure matches onto itself



Point  
– has rotational symmetry of 180°

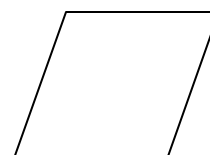


### Caution!

Parallelogram – point symmetry only!!!



Rhombus – line symmetry & point symmetry!!!



- \*slopes of parallel lines are equal
- \*slopes of perpendicular lines are negative reciprocals; product is -1

Vertical Lines:

Slope is undefined  
Equation is  $x = \#$

Horizontal Lines:

Slope is 0  
Equation is  $y = \#$

### G.3 – Angle Relationships

#### **Congruent Angles**

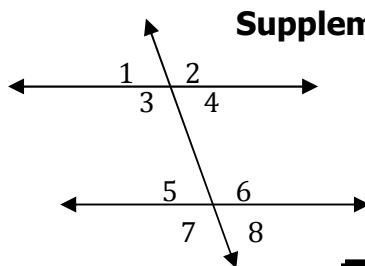
\*(If lines are parallel)

Vertical  $\angle 2 \cong \angle 3$

Alternate Interior  $\angle 4 \cong \angle 5$

Alternate Exterior  $\angle 1 \cong \angle 8$

Corresponding  $\angle 3 \cong \angle 7$



**Supplementary – sum of two angles is 180**

Linear pair

$$m\angle 5 + m\angle 7 = 180^\circ$$

Consecutive Interior

$$m\angle 3 + m\angle 5 = 180^\circ$$

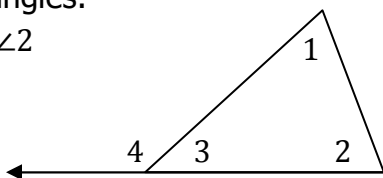
#### **Triangles**

Sum of interior angles is 180

$$m\angle 1 + m\angle 2 + m\angle 3 = 180^\circ$$

Measure of an exterior angle is equal to the sum of its two remote interior angles.

$$m\angle 4 = m\angle 1 + m\angle 2$$



#### **G.4 Ways to prove lines are parallel**

- Alternate interior angles are congruent
- Corresponding angles are congruent
- Consecutive interior angles are supplementary
- The two lines are perpendicular to the same line

### **G.5 Congruent and Similar Triangles**

#### **Congruent Triangles**

- Corresponding angles are congruent
- Corresponding sides are congruent
- Ways to prove triangles are congruent
  - SSS, SAS, ASA, AAS, HL (for right triangles)

#### **Similar Triangles**

- Corresponding angles are congruent
- Corresponding sides are proportional
- Ways to prove triangles are similar
  - AA~, SSS~, SAS~

#### **G.6 Triangle Inequalities**

To form a triangle, sum of smaller two lengths must be greater than the largest

$$L > S + M \quad \text{triangle}$$

$$L = S + M \quad \text{flat}$$

$$L < S + M \quad \text{gap}$$

Largest angle is opposite largest side, smallest angle is opposite smallest side

Base angles of an isosceles triangle are congruent

Sides opposite congruent angles are congruent

To find the possible lengths for the third side given the other sides:  
subtract given #'s  $< x <$  add given #'s

#### **G.7 Right Triangles**

Pythagorean Theorem  $c^2 = a^2 + b^2$

\*Used when two sides of a right triangle are given

Converse of the Pythagorean Theorem

\*Identify a triangle as right, obtuse, or acute

$$c^2 = a^2 + b^2 \quad \text{Right}$$

$$c^2 > a^2 + b^2 \quad \text{Obtuse}$$

$$c^2 < a^2 + b^2 \quad \text{Acute}$$

\*Check to see if triangle is possible

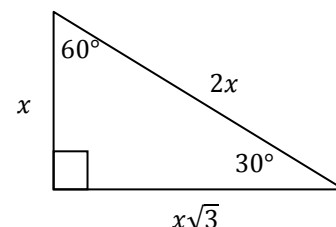
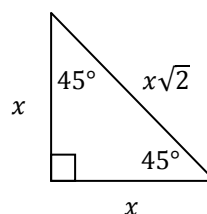
Right Triangle Trigonometry – SOH CAH TOA

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}, \quad \cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}, \quad \tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

Special Right Triangles: only when given angle

$$45 - 45 - 90$$

$$30 - 60 - 90$$



## G.8 Quadrilaterals

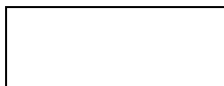
### **Parallelogram**

Opposite sides are parallel  
Opposite sides are congruent  
Opposite angles are congruent  
Consecutive angles are supplementary  
Diagonals bisect each other



### **Rectangle**

Parallelogram with:  
All right angles  
Diagonals are congruent



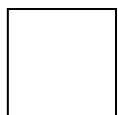
### **Rhombus**

Parallelogram with:  
Four congruent sides  
Four congruent sides  
Diagonals are perpendicular  
Diagonals bisect opposite angles



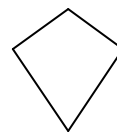
### **Square**

Parallelogram  
Rectangle  
Rhombus  
\*all 10 properties listed above



### **Kite**

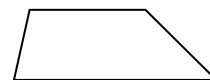
2 pair of adjacent sides are congruent  
No opposite sides are congruent



### **Trapezoid**

Exactly one pair of opp. sides parallel  
Median joins midpoints of legs and is parallel to bases

$$m = \frac{1}{2}(b_1 + b_2)$$



### **Isosceles Trapezoid**

Legs are congruent  
Pairs of base angles are Congruent  
Diagonals are congruent



## G.9 Polygons

Formulas:

$(n - 2)180$	Sum of interior angles
$\frac{(n-2)180}{n}$	Each interior angle (regular)
360	Sum of exterior angles
$\frac{360}{n}$	Each exterior angle (regular)

\*Exterior angle + interior angles = 180

\*Exterior angle and its interior angle are supplementary

### **Tessellation Information**

\*Each vertex must have a sum of 360 degrees

Regular polygons that tessellate:

- Triangle – each angle measures  $60^\circ$
- Square – each angle measures  $90^\circ$
- Hexagon – each angle measures  $120^\circ$

Combinations of regular polygons that tessellate

- square and octagon
- square and triangle
- triangle and hexagon

Other common regular polygon measurements (do not tessellate)

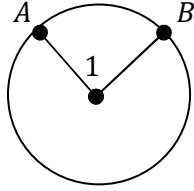
- Pentagon – each angle measures  $108^\circ$
- Octagon – each angle measures  $135^\circ$

\*\*Non-regular figures can tessellate. Make sure that the sum of the angles at any vertex add to 360

## G.10 Circles

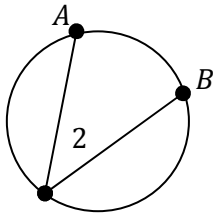
### Angles & Arcs

#### Central Angle



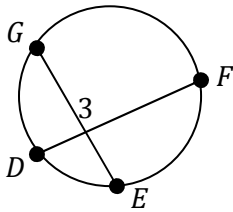
$$m\angle 1 = m\widehat{AB}$$

#### Inscribed Angle



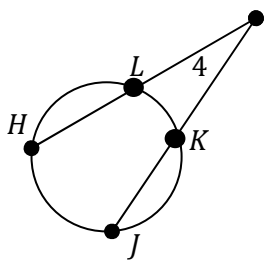
$$m\angle 2 = \frac{1}{2} m\widehat{AB}$$

#### Vertex inside circle



$$m\angle 3 = \frac{1}{2} (m\widehat{DE} + m\widehat{FG})$$

#### Vertex outside circle

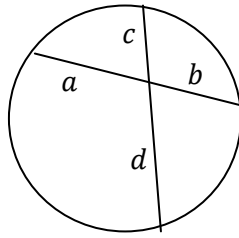


$$m\angle 4 = \frac{1}{2} (m\widehat{HJ} - m\widehat{LK})$$

### Segments

#### Two Chords

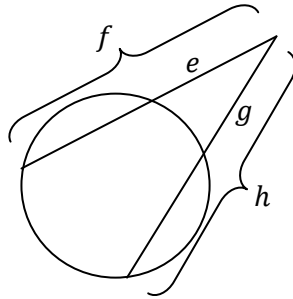
(product of segments from one chord = product of segments from the other)



$$ab = cd$$

#### Two Secants

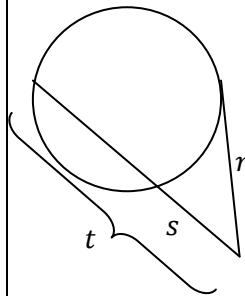
(outer secant segment<sub>1</sub> x whole secant<sub>1</sub> = outer secant segment<sub>2</sub> x whole secant<sub>2</sub>)



$$ef = gh$$

#### Tangent and Secant

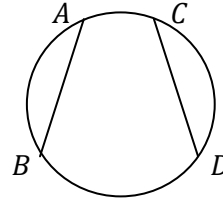
Tangent<sup>2</sup> = outer secant segment x whole secant



$$r^2 = st$$

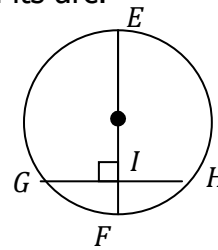
### Miscellaneous Topics

Congruent chords have congruent arcs



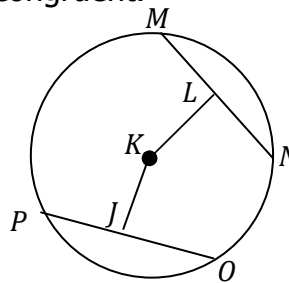
$$\overline{AB} \cong \overline{CD} \leftrightarrow \text{arc} AB \cong \text{arc} CD$$

A diameter perpendicular to a chord bisects the chord and its arc.



$$\begin{aligned} \overline{EF} \perp \overline{GH} &\leftrightarrow \overline{GI} \cong \overline{IH} \\ &\leftrightarrow \text{arc} GF \cong \text{arc} HF \\ &\leftrightarrow \text{arc} EG \cong \text{arc} EH \end{aligned}$$

Chords equidistant from the center are congruent.

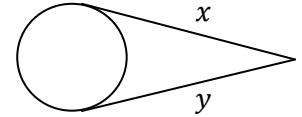


$$\begin{aligned} KL = KJ &\leftrightarrow \overline{MN} \cong \overline{PO} \\ &\leftrightarrow \text{arc} MN \cong \text{arc} PO \end{aligned}$$

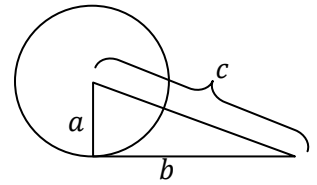
### Miscellaneous Topics

Tangents from the same exterior point are congruent.

$$x = y$$

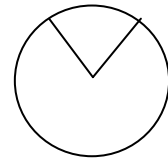


Tangent is perpendicular to the radius drawn to the point of tangency.



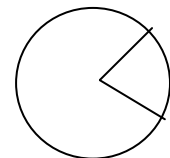
$$c^2 = a^2 + b^2$$

#### Arc Length



$$\frac{\text{degree}}{360} \cdot 2\pi r$$

#### Area of a sector



$$\frac{\text{degree}}{360} \cdot \pi r^2$$

## G.13 Lateral Area, Surface Area & Volume of 3-D Figures

**Lateral Area** – does not include base areas (ex: toilet paper roll, b-day party hat)

**Surface Area** – does include base areas (ex: soda can, closed box)

**Volume** – amount filled inside 3-D figure (ex: soda in a can, helium in a balloon)

## G.14 Proportions in similar figures

**Scale Factor**  $a:b$

**Perimeter ratio**  $a:b$

**Any Area ratio**  $a^2:b^2$

**Volume ratio**  $a^3:b^3$

Think about the measurement units for perimeter, area, and volume to help you remember the power of the ratio