Algebra II

SOL Folder Part 2

Functions

Functions

Functions

AII.6

The student will recognize the general shape of function (absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic) families and will convert between graphic and symbolic forms of functions. A transformational approach to graphing will be employed. Graphing calculators will be used as a tool to investigate the shapes and behaviors of these functions.

Notes and Formulas:

Functions: y = |x|

$$y = |x|$$

Absolute Value

If positive, goes up. If negative goes down.

$$y = \sqrt{2}$$

Square Root "Rocket"

Has a definite starting point.

$$y = x$$

Quadratic "U" shape Can go up or down

Exponential "x in exponent"

$$y = 2^x$$
 Exponential $y = \log x$ Logarithmic

Inverse of exponential equation

$$y = \frac{p(x)}{q(x)}$$

Rational Function Look for Horizontal and Vertical asymptotes

(In calc – $p(x) \div q(x)$ – use parentheses)

 $y = ax^n + bx^{n-1} + cx^{n-2} + ...$ Polynomial Function – decreasing powers

x² equation: up/down or down/up Two movements (Parabola) x³ equation: up/down/up or down/up/down Three movements (cubic)

x⁴ equation: up/down/up/down or down/up/down/up Four movements ("W")

Transformations: Remember horizontal shifts are the opposite of the sign

y = |x-2| Shifts 2 units to the right

y = |x+2| Shifts 2 units to the left

(change within your function)

Vertical shift is at the end of your function and moves the same

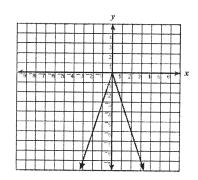
$$y = |x| + 2$$
 Shift vertex up 2

y = |x| + 2 Shift vertex up 2 y = |x| - 2 Shift vertex down 2

Calculator: You can sketch every function in your calculator using y= and zoom 6 or zoom 4

Practice Problems:

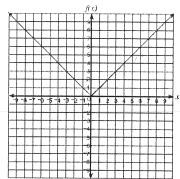
1. The graph below is an example of which type of function?



- A Absolute Value
- Exponential
- Linear
- **D** Quadratic

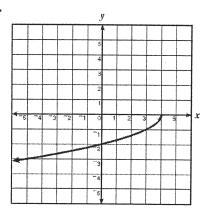
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2. Which type of function is shown?



- F Absolute Value
- **G** Exponential
- H Linear
- J Quadratic

___3.



Which most likely represents the equation

of the graph above?

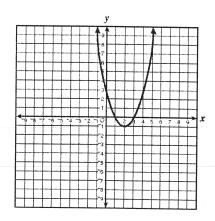
$$\mathbf{A} \quad y = \sqrt{4 - x}$$

B
$$y = -\sqrt{4-x}$$

c
$$y = -\sqrt{4 + x}$$

$$\mathbf{D} \quad y = \sqrt{4 + x}$$

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Which of the following is most likely the equation graphed above?

F
$$y = (x+2)^2 + 1$$

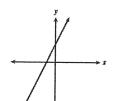
G
$$y = 5(x-1)^2 - 2$$

H
$$y = (x-2)^2 + 2$$

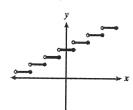
J
$$y = (x-2)^2 - 1$$

___5. Which of the following sketches could represent the graph of y = |x - a|?

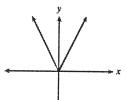
Δ



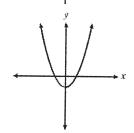
В



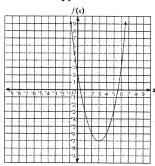
C



D



___6. Which type of function is shown?



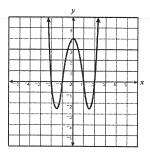
F Absolute Value

G Exponential

H Linear

J Quadratic

___7.



This is a portion of the graph of a polynomial function. If written in order of descending powers, which could be the first term of the polynomial?

A x^2

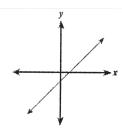
 $\mathbf{B} \quad x^3$

 $\mathbf{C} \quad x^4$

 $\mathbf{D} \quad x^5$

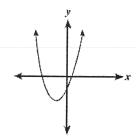
___8. Which could be a graph of $y = ax^3 + bx^2 + cx + d$ if a, b, c, and d

F

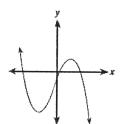


are real numbers and a < 0?

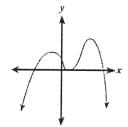
G



Н



J



All.7 The student will investigate and analyze functions algebraically and graphically. Key concepts include

- a) domain and range, including limited and discontinuous domains and ranges;
- b) zeros:
- c) x- and y-intercepts:
- d) intervals in which a function is increasing or decreasing;
- e) asymptotes:
- f) end behavior:
- q) inverse of a function; and
- h) composition of multiple functions.

Graphing calculators will be used as a tool to assist in the investigation of functions.

Notes and Formulas:

Domain: x values "alpha order"

To be a function: No two x's are the same or it passes

Range: y values

Vertical Line Test

Zero (root, solution, x-intercept) of a function: Where your function has a value of 0 (where your graph crosses the x-axis)

Calculator: Put eqt. in y= Zoom 6 or 4

2nd Calc #2 – zero Left Bound/Right Bound/Guess

Work backwards! Substitute the answers given to see which answer gives you a 0

To find value of a function: Plug the number into your expression for every x

To find composite functions: f(g(x)) Substitute the entire inner function into the outer function in the x position

To find inverse of a function: 3 steps - 1. y =

2. switch x and y

3. Solve for y Inverse ($f^{-1}(x)$) will be what y =

On a graph: x intercept – where your function crosses the x-axis

v intercept – where your function crosses the y-axis

turning point - point where your function turns the corner and switches direction

interval increasing - region on graph where y values are increasing interval decreasing - region on the graph where y values are decreasing

Asymptotes: Vertical & Horizontal Asymptotes – Vertical: Look at denominator (what x makes a 0)x = ?

Horizontal: same power – coeff./coeff. y = ?higher power below – at y = 0

End Behavior: Look at your graph and your question. Read carefully. It will ask you what y value you

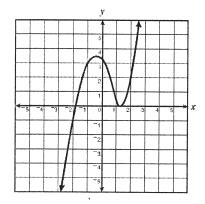
are approaching as x approaches a given value. If it is a rational function, you are

always approaching your horizontal asymptote. Use your calculator !!

Use your calculator to help you !!!!

Practice Problems:

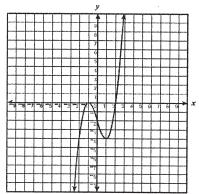
___1.



This is a portion of the graph of a polynomial function. Apparently the function has a turning point at -

- **A** (-2,0)
- **B** (0,3)
- **C** (-.5,3.5)
- D It does not have one

___2.



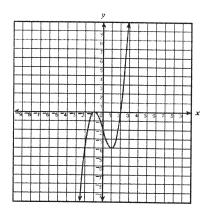
This is a portion of the graph of a polynomial function. Apparently the function has a domain of –

- F all real numbers
- $G \times > 0$
- H y > 0
- J 9 < x < 10

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3. TEQ



The graph of a polynomial function is graphed above. Click and Hi-light each expression that represents an increasing interval.

(You just check every one)

$$-\infty < x < \infty$$

$$-\infty < x < -1$$

$$1 < x < \infty$$

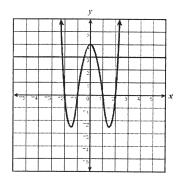
$$-3 < x < \infty$$

$$-\infty < x < 0$$

___4. Which describes the End Behavior of the function $f(x) = x^3 - 4x^2 + 4x$ as x approaches infinity?

- F y approaches infinity
- G y approaches 0
- H y approaches negative infinity
- J y approaches 2

5.



This is a portion of the graph of a polynomial function. This function apparently has the following types of roots -

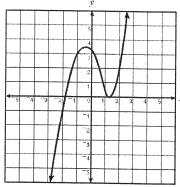
- A 5 different real roots
- B 4 different real roots
- C 2 real roots and 2 imaginary roots
- D no real roots

___6. Which value is not a zero of

$$P(x) = x^3 + 3x^2 - x - 3$$
?

- F 1
- G -1
- **H** 3
- J -3

____7.



This is a portion of the graph of a polynomial function. Apparently the function has a double zero —

- A between -2 and -1
- B between -2 and 1
- C between 1 and 2
- D between 3 and 4

____8. The polynomial function

$$y = x^3 - 3x^2 + x + 1$$

has a zero between -

- F -4 and -3
- G -2 and -1
- H -1 and 0
- **J** 3 and 4

____9. If the domain of $f(x) = 2x^2 - 3$

is limited to $\{-3, -1, 1, -3\}$, what is the range?

- **A** {-21, -5, -1, 15}
- **B** {-21, 15}
- **C** {-1, 15}
- **D** {1, 5, 15, 21}

__10.
$$f(x) = x^2 - 2x$$

$$g(x) = x - 3$$

Which of the following expressions

represents g(f(x))?

F
$$x^3 - 5x^2 + 6x$$

G
$$x^2 - 2x - 3$$

H
$$x^2 - 3x - 3$$

J
$$x^2 - 8x + 9$$

___11. If $f(x) = 5x^2 - 7$, what is f(-3)?

- **A** -52
- **B** -22
- **C** 38
- **D** 45

___12. Which function represents the inverse of the function f(x) = x + 2?

$$f^{-1}(x) = x - 2$$

G
$$f^{-1}(x) = -x - 2$$

H
$$f^{-1}(x) = -x + 2$$

J
$$f^{-1}(x) = -(x+2)$$

13. TEQ: Click and drag the horizontal and vertical asymptotes for the function

$$g(x) = \frac{x-1}{x-4}.$$

You just select and put the answer in each box.



Vertical

Horizontal

y = 1

x = 1

x = 4y = 4

x = -4

AII.8

The student will investigate and describe the relationships among solutions of an equation, zeros of a function, x-intercepts of a graph, and factors of a polynomial expression.

Notes and Formulas:

Solutions of equation, zeros of a function, and x-intercepts all refer to the same thing: Where is the value of my function equal to zero?

If zeros are a,b,c Then factors are (x-a)(x-b)(x-c) **** Factors will contain opposite sign of roots

Practice Problems:

- ___1. Which is a zero of $f(x)=x^2 + x 6$?
- **A** -3
- **B** -2
- C 0
- **D** 3
- ____2. A polynomial function has a zero at

x = -4. Which expression *must* be a

factor of the polynomial?

- F x-4
- Gx-2
- Hx+2
- J x + 4
- ___3. Which of the following functions has

x-intercepts at -2 and 1?

A
$$y = x^2 - x - 2$$

B
$$y = x^2 + x - 2$$

C
$$y = x^2 - 2x + 1$$

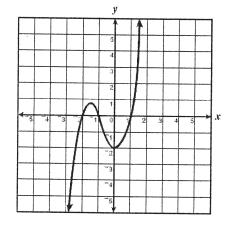
D
$$y = 2x - 1$$

____4. Which of the following sets contains

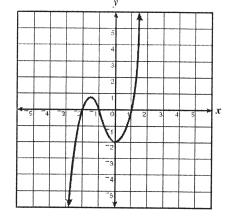
all the apparent zeros for the function

shown?

- F {1}
- **G** {-2, 0, 2}
- **H** {-2, -1, 1}
- J {-2, -1, 0, 1}



____5. A section of the graph of a polynomial function with integral coefficients is shown. Which of the following sets most likely contains only elements that are factors of the polynomial?



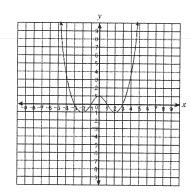
A
$$\{(x-2), (x-1.5)\}$$

B
$$\{(x-2), (x-1), (x+1)\}$$

C
$$\{(x+2), (x+1), (x-1)\}$$

D
$$\{x, (x-2), (x-1), (x+1)\}$$

____6. Which of the following sets contains all the apparent zeros for the function shown?



Statistics

Statistics

The student will investigate and apply the properties of arithmetic and geometric AII.2 sequences and series to solve real-world problems, including writing the first n terms, finding the $m{n}^{ ext{th}}$ term, and evaluating summation formulas. Notation will include $\sum^n X_i$ and a_{n}

Notes and Formulas:

Formulas will be on your Formula Sheet

Pay attention to whether you have an arithmetic (Adding) or a geometric (Multiplying) sequence

If you get stuck with the formulas - you can always just write all of your numbers out and find a term or add the numbers up to get the sum

Arithmetic Term

Arithmetic Sum

Geometric Term

Geometric Sum

Infinite Geom, Sum

$$A_n = a_1 + (n-1)d$$

$$S_n = \frac{n(a_1 + a_n)}{2}$$

$$A_n = a_1 \bullet r^{n-1}$$

$$A_n = a_1 + (n-1)d$$
 $S_n = \frac{n(a_1 + a_n)}{2}$ $A_n = a_1 \cdot r^{n-1}$ $S_n = \frac{a_1(1 - r^n)}{1 - r}$

$$S_{\infty} = \frac{a_1}{1 - r}$$

Practice Problems:

___1. What is the sum of the series

___3. If $a_n = 2^{n-1}$, Which number

defined by $\sum_{n=0}^{4} (3-2n)$?

A -5

 \mathbf{D} 0

represents a_4 = ?

A 15

B 8

C 7

D 6

2. Two geometric means between 2 and 54 are -

4. What are two arithmetic means

between 3 and 24 ?

F 8 and 12

G 8 and 16

H 9 and 16

J 10 and 17

F 4 and 12

G 6 and 12

H 6 and 18

J

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TEQ 5. Driving a piling into a harbor bottom, a pile driver sinks the piling 24 inches on the first stroke, 18 inches on the second stroke, and $13\frac{1}{2}$ inches on the third stroke. If the sequence is continued, how far will the piling be driven down on the 5th stroke?

Put your answer in the box.



- ___6. What is the value of $\sum_{n=1}^{6} 2^n$?
- **F** 62 **G** 126
- H 128 J 252
- ____7. If $a_n = 1 + \frac{1}{n}$, then what is a_9 ?
- **A** $\frac{11}{10}$
- **B** $\frac{10}{9}$
- $c = \frac{9}{8}$
- **D** $\frac{3}{2}$

All.9 The student will collect and analyze data, determine the equation of the curve of best fit, make predictions, and solve real-world problems, using mathematical models. Mathematical models will include polynomial, exponential, and logarithmic functions.

Notes and Formulas:

If given a graph, LOOK at the shape of your graph to determine if the Best Model would be Linear, Quadratic, Cubic, Exponential, or Logarithmic

Linear - A line

Quadratic - Parabola "U" shape

Cubic - "Egyptian"

Exponential – One side is increasing rapidly

Logarithmic - Inverse of Exponential

If given a set of data, read carefully. If it states the type of model, then you are lucky and you use it.

If it states to determine the best model for the data, then do a rough sketch on your paper and you look to see which model is best

Calculator: Remember to put x's in List 1 and y's in List 2. Clear anything in your lists.

STAT - CALC - #4 Linear

- Enter

#5 Quadratic

#6 Cubic

#9 Ln (Logarithmic)

#0 (10) Exponential

Practice Problems:

___1. In 1940 the life expectancy

at birth in the general public was

62.9 years. By 1980 it had risen to

73.7 years. Assuming a linear relation,

which is the *best* prediction of life expectancy

in the year 2000?

A 76.4 **B** 79.1

C 79.9 **D** 84.5

___2. Which is the best quadratic model for

the data below?

{(-1,2), (0,1), (2,5)}

F $y = x^2$

G $y = x^2 + 1$

 $H y = 2x^2 + x + 1$

 $J v = x^2 + 2x + 1$

___3. Which type of function would best

model the data below?

Х	У
0	2
4	6
8	14
12	26
16	54
20	110

- **A** Quadratic
- **B** Cubic
- **C** Exponential
- **D** Logarithmic
- ____4. Using #3 above, what is your

Best Model equation for the data?

F
$$y = 2.21x+1.20$$

G
$$y = 2.21(1.20)^x$$

H
$$y = 1.20(2.21)^x$$

J
$$y = 2.21x^2 + 1.20$$

____5. Using #3 and #4 above, what is your

best prediction for when x = 30?

- **A** 454
- **B** 500
- C 525
- **D** 550

____6. The table below shows the number of students enrolled in the Algebra Trig program at York High School the first 5 years the course was begun.

Year	No. of
(x)	students
	(y)
1	55
2	71
3	84
4	97
5	108

Which of the following equations most closely describes the relationship between y, the number of students enrolled, and x, the number of years the class has existed?

$$F y = x + 13$$

$$G y = 13x + 43$$

$$H y = 10x + 23$$

J
$$y = 13x - 43$$

- ____7. Using the data for #6 above, what is the best prediction for the year 10 ?
- A 100
- **B** 150
- C 175
- **D** 225

All.10 The student will identify, create, and solve real-world problems involving inverse variation, joint variation, and a combination of direct and inverse variations.

Notes and Formulas:

Direct Variation:
$$\frac{y}{x} = k$$

k is your constant of proportionality

Direct=Divide

Inverse Variation:
$$yx = k$$

Inverse=Multiply

Joint Variation: Divide by both

Practice Problems:

___1. The time it takes to travel a given

distance varies inversely as the

average rate of travel. Averaging

42 miles per hour, it takes John

5 hours to drive to Pittsburgh. If it took

him 4 hours and 20 minutes to reach

Pittsburgh on his last trip, what was his

average rate of travel?

A 36.4 mi./hr

B 46.7 mi./hr

C 48.5 mi./hr

D 49.4 mi./hr

____2. The volume (V) of a sphere varies directly with the cube of its radius (r).

If ${\it k}$ is the constant of proportionality,

which is the formula for this

relationship?

 \mathbf{F} V = kr

 $\mathbf{G} \quad V = kr^3$

 $H V = \frac{k}{r^3}$

 $r = kV^3$

- ____3. Hooke's law states that the force required to stretch a spring varies directly with the distance the spring is stretched. If a 10-pound force stretches a spring 2 inches, what force is required to stretch the spring 5 inches?
- A 15 pounds
- B 20 pounds
- C 25 pounds
- D 30 pounds
- ____4. The amount of interest (*I*) owed on a loan varies directly with the length of time (*t*) of the loan. If *k* is the constant of proportionality, which formula represents this relationship?
- F I = kt
- $G I = \frac{k}{t}$
- $\mathbf{H} t = kI$
- $J \quad t = \frac{k^2}{I}$

- _____5. Boyle's Law states that, for a fixed amount of gas, the volume of the gas at a constant temperature is inversely proportional to the pressure. If a certain gas occupies 9.84 liters at a pressure of 50 centimeters of mercury (cm Hg), what is the approximate pressure when the volume is increased to 12 liters?
- A 39.8 cm Hg
- **B** 41.0 cm Hg
- C 43.2 cm Hg
- **D** 45.1 cm Hg
- ___6. In which of the following is z directly proportional to x and inversely proportional to the square of y?

$$\mathbf{F} \quad z = k \frac{x^2}{y}$$

$$\mathbf{G} \quad z = kxy^2$$

$$H \quad z = \frac{x}{y^2} k$$

$$\mathbf{J} \quad z = k \frac{y}{x}$$

All.11 The student will identify properties of a normal distribution and apply those properties to determine probabilities associated with areas under the standard normal curve.

Notes and Formulas:
Formulas will be on your formula page.
You must be able to draw your standard normal curve and know your percentages.
If you land on a marker, you can use your percentage from your curve.
If you are between markers you must use your z-score and your table or calculator.
$z-score = \frac{x-\mu}{\sigma}$ x=value, μ = mean of your data, σ = standard deviation
Look at your formula page !!!
Practice Problems:
1. TEQ: Roanoke had the following amounts
of Snowfall last January:
4 2" 2 3" 6" 7 9" 10" 5 5" 12 5" o"

Find the mean and the standard deviation. Place

your answers in the box.

Mean	Standard Deviation

2. At Thomas Nelson, the Pre-Test for Mathematics has 200 points on the test. The mean is 120 and the standard deviation is 20. What percent of students score below 100 points?	4. On his midterm exam, Jimmy scored 75 points, which was exactly 2 standard deviations above the mean. If the standard deviation for the test is 4, what is the mean for the test?
F 50%	F 79
G 34%	G 75
H 16%	H 71
J 7%	J 67
3. In order to pass the Mathematics Pretest a student must score 140 points. If 360 freshmen took the pre-test last fall, how many passed? (Use #2 above)	5. Susie's test grades in Algebra 2 for the third quarter are 100, 80, 60, 88, 92, and 90. How many scores are within one standard deviation of the mean?
A 200	A all 6
B 240	B 5
C 302	C 4
D 324	D 3
	6. TEQ: A survey of 20 colleges found that seniors graduated with an average \$12,000 in debt from student loans. The debt was normally distributed with a standard deviation of \$3200. Find the probability that a senior graduated owing more than \$16,000. Place your answer in the box.

All.12 The student will compute and distinguish between permutations and combinations and use technology for applications.

Notes and Formulas:	
Use your calculator! Math → PRB Permutation: Order Matters Formula on formula Combination: Order does not matter Formula on Practice Problems:	
riactice Fioblems.	
1. In an Algebra II class there are	3. TEQ: How many permutations can be
8 A's, 6 B's, 5 C's, and 2 D's, and 4 F's.	formed using the letters in the word
Susie is in the class. What is the	MATHEMATICS ?
probability Susie got an A?	
A 8/25	Put your answer in the box.
B 8/16	CONTRACTOR
C 8/21	
D 50%	
2. If digits can be repeated, how many	4. If numbers and letters can be
3-digit numbers can be formed using the	repeated, how many different 6-digit
digits 1, 2, 3, and 4 ?	license plates can be made if the first two
F 24	positions are letters and the last four are
G 48	digits?
H 64	F 492.804
J 256	G 676,000
	H 6,760,000
	J 455,625

___5. TEQ: A committee of 3 teachers and 3 students is to be formed to judge a contest. If there are 7 students and 5 teachers to choose from, how many different committees could be formed? Put answer in the box. ____6. Meredith has 14 girls on her softball team. She wants to have 2 co-captains. How many different choices does she have? $\mathbf{F}_{14}P_{2}$ **G** 14! H ₁₄C₂ J 14 ____7. A normally distributed set of 968 values has a standard deviation of 11 and a mean of 108. Which is closest to the number of values expected to be above 125? **A** 910 **B** 989 C 210 **D** 59